

Bumpy black holes from spontaneous Lorentz violation

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ABSTRACT: We consider black holes in Lorentz violating theories of massive gravity. We argue that in these theories black hole solutions are no longer universal and exhibit a large number of hairs. If they exist, these hairs probe the singularity inside the black hole providing a window into quantum gravity. The existence of these hairs can be tested by future gravitational wave observatories. We generically expect that the effects we discuss will be larger for the more massive black holes. In the simplest models the strength of the hairs is controlled by the same parameter that sets the mass of the graviton (tensor modes). Then the upper limit on this mass coming from the inferred gravitational radiation emitted by binary pulsars implies that hairs are likely to be suppressed for almost the entire mass range of the super-massive black holes in the centers of galaxies.

KEYWORDS: Black Holes, Space-Time Symmetries.

Contents

1. Introduction and summary	1
2. No-hair theorems	8
3. Higgs phases of gravity	12
3.1 Setup and its basic properties	12
3.2 Phenomenology of massive gravitation	15
4. Black hole backgrounds	17
4.1 Ghost condensate	18
4.2 Non-linearities and the simplest black holes in massive gravity	19
5. The toy instantaneous QED model	21
5.1 Lorentz violating electrodynamics in flat space	21
5.2 Covariant action	23
5.3 Black Holes	23
6. Instantaneous interactions and black hole hairs	25
6.1 Instantaneous scalar field	25
6.2 Lorentz-violating QED	28
7. Hair nurturing	30
8. Concluding remarks	33
A. UV modifications of gravity and black holes	34
B. Electric dipole hair	35

1. Introduction and summary

Gravity remains the most mysterious force in nature, affecting properties of space and time at the most fundamental level. Large quantum fluctuations of the metric at the Planck scale indicate that the very basic principles of quantum field theory, such as locality, are likely to be drastically modified in quantum gravity. Unfortunately, there is little hope to directly probe gravity in this regime.

The existence of black holes provides an alternative window to explore non-perturbative gravitational dynamics at energy densities well below the Planck scale. Indeed, for a long time black holes have been a principal “theoretical laboratory” for quantum gravity. Many

of the advances in string theory resulted from attacking the fundamental puzzles of black hole thermodynamics. Moreover, black holes hide a singularity in their interiors that probes the quantum gravity regime. Unfortunately, “cosmic censorship” appears to hide from our view what is happening at the singularity.

In practice General Relativity (GR) is routinely used to describe gravitational phenomena that span a wide range of scales from the Solar system to the entire Universe. GR has survived many precision tests in the Solar system and has successfully predicted the emission of gravitational waves by binary pulsars (see, eg., [1]). However, most of the existing tests concern the weak field regime. The tests of GR in the strong field limit, i.e., when the non-linearities of the Einstein equations play an essential role, are more difficult to obtain.

One such case is the dynamics of the Universe as a whole. The cosmological model based on GR is confirmed by observations with an ever-growing precision [2] up to the nucleosynthesis epoch. This model requires, however, that the present Universe is dominated by a dark energy and dark matter of as yet unknown origin. It is not clear, therefore, to what extent the above agreement can be considered as a confirmation of GR itself.

Another possibility to study non-linear effects of gravity is provided by the astrophysical black holes that are expected to be a perfect laboratory for quantitative tests of GR in the strong field limit. It is conceivable that in the near future the validity of the Schwarzschild or Kerr metric around astrophysical black holes could be tested with high precision through a variety of astronomical observations.

There exists two techniques which may allow to reconstruct the metric of the black hole close to the horizon. The first one consists of observing the electromagnetic radiation which comes from the innermost region of the accretion disk of a black hole and encodes information about the space-time structure of that region. Extracting this information requires detailed modeling of the accretion disk in order to disentangle the physical effects which depend on the structure of the metric (such as strong gravitational lensing, large redshifts and time delays) from other unknowns such as the details of the physical state of the gas in the disk, the accretion rate or the mechanism responsible for the emission of the electromagnetic radiation (eg. [3]). The GR effects influence the emission coming from different parts of the disk in different ways, sometimes leading to “easily” identifiable features in the light curve (see, e.g., [4]). For a recent review of astrophysical black holes and the various observational techniques to characterize them see [5].

Another technique being developed for future gravitational wave observatories relies on the detailed study of the time dependence of the emitted radiation during the inspiral and merger phases of binary systems involving at least one black hole. For the purpose of testing GR the most promising candidates are perhaps the so-called extreme mass ratio inspirals (EMRI), that is compact stellar mass objects captured by supermassive black holes ($\sim 10^6 M_\odot$) in galactic nuclei. Because of the large mass ratio the small compact object is to a very good approximation a test particle orbiting around a black hole. LISA is expected to detect about a hundred of such EMRIs per year. Because of the small mass ratio the inspiral should be observable by LISA for years, during roughly 10^5 orbits. The study of such events could lead to strong constraints on the multipole moments of the

space-time around the black hole and thus provide a precision test of the black hole metric.

A point important for the purpose of this paper is that the multipole moments of the black hole metric are very sensitive to potential deviations from GR. Indeed, a striking property of black holes in GR is the absence of “hairs” [6]–[10], namely no matter what the shape of the collapsing object is, all multipole moments of the resulting black hole are determined just by its total mass and angular momentum. In fact, this no-hair property is one of the main features which distinguishes black holes from ordinary massive non-radiating objects. With LISA one expects to be sensitive to 6-7 lowest black hole multipoles with a precision at the level of a few percents, and thus be able to obtain a quantitative verification of the universality of the black hole metric (the absence of “bumps”) [11].

Given the observational promise and the considerable efforts put to measure the detailed properties of the astrophysical black holes, one may wonder what are the benchmark theoretical models which provide predictions for these observations different from those of GR. In particular, what would be the implications of the black hole bumps, if they were to be discovered?

Naively, one might expect that a black hole is the most natural place to test alternative theories of gravity. However, the actual situation is more complicated. One of the problems is that the space-time curvature around astrophysical black holes is very small. Consequently, black hole observations have practically no chance to discover short distance modifications of gravity such as those induced by higher dimensional operators in the gravitational action, or those due to large extra dimensions. Indeed, the sizes and the curvature radii of the astrophysical black holes are at least of the order of few kilometers (for the stellar mass black holes), while the existing short-distance tests of the gravitational force do not find any deviations from GR up to distances as short as a fraction of a millimeter [12, 13]. As a result, even the most extreme scenarios with the ultra low quantum gravity scale [14] do not lead to measurable changes in the properties of astrophysical black holes due to UV effects in the range of parameters where they are compatible with the direct gravity tests. In appendix A we make this argument more quantitative.¹

As an alternative to the short distance effects, one is naturally lead to models that modify gravity at large distances. Recently, there has been a revival of interest in long-distance modifications of gravity which was to a large extent motivated by the observation of the accelerated expansion of the Universe. Though no compelling alternative to the simplest Λ CDM scenario has emerged so far, these efforts resulted in a much clearer theoretical understanding of the possible models, their characteristic features and potential observational signatures.

To find the most promising class of theories that could be tested with black hole

¹Note that it was suggested [15, 16] that in the Randall-Sundrum model (that can also be thought of as a modification of gravity at short scales) the evaporation rate of the black holes localized on our brane can be significantly enhanced due to the presence of the continuum spectrum of the light Kaluza-Klein modes. This may lead to the rapid evaporation of the astrophysical black holes with masses of order few Solar masses. This proposal still remains somewhat controversial [17], but even if true it does not predict anything new for the observations of the space time around astrophysical black holes, it just shortens their lifetime.

observations, let us start with the brief overview of the modified gravity theories. Probably the best studied class of long distance modifications of gravity are scalar tensor theories of the Brans-Dicke type. The so called $f(R)$ modifications of gravity also belong to this category in their simplest versions [18]. Here the long-distance effects are due to the presence of a new light scalar degree of freedom. As we will discuss shortly, the “no-hair” theorems imply that the study of black holes properties is not a promising way to constrain these models.

Another class of theories includes Lorentz invariant Fierz-Pauli model of massive gravity [19] and brane world constructions where the four-dimensional graviton mass is replaced by a resonance with a finite width that is due to the escape of gravitons into extra dimensions [20–22]. The common theme in the study of these models is the dynamics of the longitudinal graviton polarization, which typically leads to strong coupling (and, as a result the loss of predictability) at an unacceptably low energy scale, and/or to the appearance of ghosts around curved backgrounds [23]–[29].

A notable exception is the five-dimensional Dvali-Gabadadze-Porrati (DGP) brane world model where non-linear effects provide an extra contribution to the kinetic term of the longitudinal mode of the right sign which prevents strong coupling in the vicinity of the sources [30] (unfortunately, this contribution has a wrong sign for the cosmologically most interesting self-accelerating branch, and the perturbative analysis reveals a ghost in the spectrum of linear perturbations around this branch [29, 31, 32]).

For many purposes the DGP model can be thought of as a very peculiar scalar-tensor theory where the derivative scalar self-interaction results in the “chameleon” or self-shielding behavior near massive sources (cf. [33]). This shielding is crucial for any such theory not to be already ruled out by the Solar system tests, in particular, by the deflection of light measurements. It results in interesting non-linear effects at short enough distances from a massive source giving rise, for instance, to a small anomalous precession of the Moon perihelion. This effect is potentially observable by the next generation of the Lunar ranging experiments [34, 35]. Yet another striking result of the non-linearities is the possibility for the superluminal propagation in certain backgrounds [36].

Finally, there exists a family of models which may be regarded as the “Higgs phases” of gravity in which Lorentz invariance is spontaneously broken by condensates of scalar fields. The breaking of Lorentz invariance that differentiates these models from the ones discussed previously is essential to avoid the problems of strong coupling and ghosts that plague the Lorentz invariant models [37–39]. It also allows these models to avoid the constraints coming from the deflection of light without invoking the non-linearities. Examples of such models are the so-called “ghost condensate” model [37], as well as more general theories of Lorentz-violating massive gravity [39, 40]. A closely related class of models with non-trivial vacuum expectation values of the vector fields is represented by the Einstein aether/gauged ghost condensate models [41–43]. The relativistic MOND theories were also shown to belong to this category [44].

To finish this brief survey of the infrared modifications of gravity, it is worth noting that in spite of the considerable progress in constructing consistent low energy effective theories that modify gravity at long distances, none of these models (neither Lorentz in-

variant nor Lorentz violating) have so far been derived from a consistent microscopic theory. Moreover, many properties of these theories (in particular, those related to the black hole thermodynamics discussed later in the current paper) strongly suggest that if such a microscopic theory exists it is likely to be very different from string theory — the most successful candidate for a theory of quantum gravity — at least in its regimes studied so far.

Which of the above classes of models, if any, are most likely to give alternative predictions for observations of astrophysical black hole? We have already stressed that the black hole “no-hair” theorems provide a very clean set of observables sensitive to the new physics — deviations of the black hole multipole moments from their universal GR values. However, quite generically, these very theorems prevent new physics from affecting the black hole metric.

To illustrate the origin of the problem let us consider a generic model of the Brans-Dicke type, i.e., let us assume that in addition to the metric there exists a light scalar field which by Lorentz invariance should be coupled to the trace of the energy-momentum tensor. Such a field provides an extra contribution to the Newtonian $1/r$ potential between non-relativistic sources. However, it does not affect the deflection of light in the gravitational field of the Sun. Consequently, the existence of such a field would lead to a discrepancy between the values of the Solar mass deduced from the analysis of the planetary motion and from the deflection of light. This gives rise to stringent constraints on the strength of the scalar force [1].

However, if we were unlucky to have a black hole in the center of the Solar system we would never be able to obtain such bounds. The no-hair theorems state that the black hole horizon is not able to support a non-zero static profile of the scalar field. Consequently, there would be no extra force due to the scalar field and no discrepancy between the planetary motion and the deflection of light. Thus, in the case of the Brans-Dicke type models black holes turn out to be the worst (in fact, hopeless) place to distinguish the conventional Einstein gravity from a modified theory.

This example, in spite of its simplicity, actually correctly captures the nature of the obstacles for constructing models with a modified black hole metric. Also, it suggests that instead of trying to find a model where the black hole metric is just slightly modified as compared to the GR predictions, a better strategy may be to find a way to avoid the black hole no-hair theorems altogether, so that the higher multipole moments are not universal.

We will review in some detail the physics of the no-hair theorems in section 2, but already at the intuitive level it is clear that these results follow from the very generic properties of the gravitational horizons. This suggests that the best way to violate the no-hair theorems is to consider theories with spontaneous Lorentz breaking, where the causal structure can be modified as compared to the standard case.

As we will explain now, this intuition can be made precise, and very general thermodynamical considerations strongly suggest that black holes *must* have hairs if Lorentz invariance is spontaneously broken [45]. Recall that the way the conventional laws of thermodynamics are recovered in the presence of black holes in GR is truly remarkable. Indeed, one may worry that the entropy can be lost behind the black hole horizons invalidating the second law of thermodynamics. However, as first suggested by Bekenstein, it is natural to

assign to black holes an entropy proportional to the horizon area. With this assignment the net entropy of a black hole and the outer region never decreases and the second law of thermodynamics is saved.

For this proposal to be self-consistent black holes need to have temperature T_H related to the to the energy (mass) M and the Bekenstein entropy S_B in the usual way,

$$dM = T_H dS_B .$$

This is indeed true in GR with T_H being the Hawking temperature of the black hole.

To see how the black hole thermodynamics is modified in the presence of the spontaneous Lorentz breaking, note that in this case different species propagate with different maximum velocities v even in flat space [46]. Observationally, there are extremely tight bounds on the differences in maximum velocities for the Standard Model fields [47]. However, the experimental constraints are easily satisfied if the hidden sector where the Lorentz-breaking condensate develops does not have direct couplings to the Standard Model fields, apart from those generated by graviton loops. For the purpose of the argument below the absolute magnitude of these coupling is irrelevant.

Progress achieved in recent years in understanding the gravitational dynamics in the presence of spontaneous Lorentz violation made it possible to study the consequences of the velocity differences in curved space as well, and in particular in a black hole background. The main result of these studies is very simple: the effective metric describing propagation of the field with $v \neq 1$ in the Schwarzschild background has the Schwarzschild form with a different value of the black hole mass. As one could have expected, the black hole horizon appears larger for subluminal particles and smaller for superluminal ones. As a consequence, the temperature of the Hawking radiation is not universal any longer; “slow” fields are radiated with lower temperature than the “fast” fields.

This makes it impossible to define consistently the black hole entropy as being determined just by its mass and the angular momentum. Indeed, in the presence of at least two fields with different propagation velocities, it is straightforward to provide examples of processes such that the black hole mass and the angular momentum remain constant while the entropy outside decreases. One example of such a process relies on the Hawking radiation [45] (see figure 1); another is a generalization of the classical Penrose process of the energy extraction from a rotating black hole [48].

The second law of thermodynamics follows from very basic principles of quantum theory, such as unitarity, so in order to have a chance of being derived from a consistent microscopic theory Lorentz violating models have to provide a way to restore the validity of the second law in the presence of black holes. The processes described in refs. [45, 48] which reduce the entropy *outside* of the horizon also change the state *inside* the black hole. Consequently, a contradiction with the second law can be avoided provided this change is observable from outside. In other words, black holes should have hairs on top of the mass and angular momentum, which allow an observer to “monitor” their interior state, just like it is possible (at least in principle) for ordinary stars.

As we explain below, this indeed happens quite generally in the Higgs phases of gravity. Namely, a striking property of the Lorentz-violating models with a massive graviton is the

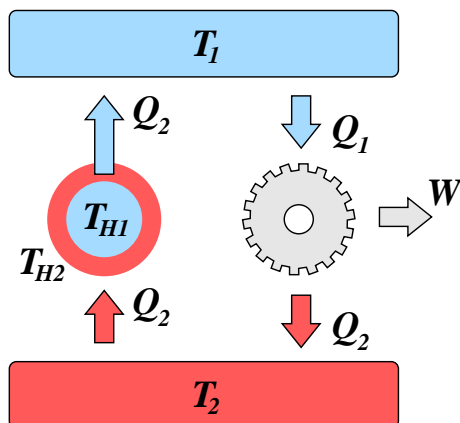


Figure 1: In the presence of the spontaneous Lorentz breaking black holes can have different temperatures for different fields. This allows to perform thermodynamic transformations whose net effect is the transfer of heat Q_2 from a cold reservoir at temperature T_2 to a hotter one at temperature T_1 (*left*). Then one can close a cycle by feeding heat Q_1 at the higher temperature T_1 into a machine that produces work W and as a byproduct releases heat Q_2 at the lower temperature T_2 (*right*). The net effect of the cycle is the conversion of heat into mechanical work.

presence of the instantaneous gravitational interactions. It is relatively easy to understand their origin. Already in the conventional GR the graviton propagator in non-covariant gauges (for instance, in the Newtonian gauge) contains pieces that give rise to the static $1/r$ potential and appear to be instantaneous. Of course, there are no physical instantaneous interactions in GR; in the non-covariant gauges this comes out as a result of the subtle cancelations between different parts of the graviton propagator. In the Higgs phase these cancelations are no longer exact, and physical instantaneous forces are present. Spontaneous breaking of Lorentz invariance introduces a preferred time in the Higgs phase and in this way the causality paradoxes usually associated with the superluminal propagation are avoided.

Given the presence of instantaneous interactions it should not be a big surprise that black holes have an infinite amount of hairs/bumps. What is interesting, is that the above thermodynamical argument strongly suggests that this should be a property of all Lorentz-violating models, excluding the “benign” possibility that there exists a finite universal maximum propagating velocity (for instance, if all fields propagate subluminally).

Apart from the Lorentz violating models, the brane world DGP model was also found to possess a superluminal mode as a result of the non-linear dynamics [36]. Its propagation velocity is background-dependent, and in principle can be arbitrarily high. This may be an indication that black holes are bumpy in the DGP model as well. Unfortunately, an explicit black hole solution which would allow the study of perturbations is not yet available in this model, so one is not able to verify whether this expectation is true or not.

The rest of the paper is organized as follows. We start with reviewing the basics of the black hole no-hair theorems in section 2. In section 3 we review the Lorentz-violating models of massive gravity. In section 4 we describe the spherically symmetric black hole

solution in these theories and some properties of the rotating black holes. In section 6 we explain how instantaneous modes that are generically present in the Higgs sector of gravity lead to the infinite amount of black hole hairs. To avoid unnecessary technicalities, instead of massive gravity we consider the Lorentz violating massive electrodynamics [49, 50], which is much simpler technically and shares with the former the relevant physical properties. We describe this theory in section 5. In section 7 we estimate the magnitude of the black hole bumps. In the minimal models it turns out to be related to the mass of the gravitational waves; the limits on the latter imply that the bumps are likely to be large only for the most massive galactic black holes (with masses of order $10^9 M_\odot$). We summarize our conclusions in section 8.

2. No-hair theorems

In order to understand how Lorentz violating models get around the no-hair theorems let us start with reviewing how they work in the conventional theories. The aim of this section is to show that in order to establish the presence of hairs one can simply look for finite energy solutions of the linearized equations in a black hole background.

As a simplest example let us consider a scalar field ϕ with mass m_ϕ . For simplicity it is convenient to consider the pre-existing, neutral with respect to the scalar field, black hole or star and ask what an external observer at the constant radius r from the object will measure if there is a small amount of scalar charge falling in.

Clearly, in case of a star an observer will be able to follow what happens with a charge by accurately measuring the scalar field profile outside. For the later purposes it is useful to formulate this somewhat more formally. Namely, the scalar field outside the star (the quantity which can be measured by the outside observer) satisfies the source free equation at late times, after the scalar charge crossed the surface of the star. The possibility of having a non-trivial scalar profile is related to the possibility of having a non-vanishing boundary conditions for a scalar field at the surface of the star, which encode the information about the fate of the charge inside.

The situation is different for a black hole in several respects. First, as seen by the outside observer, the charge never crosses the black hole surface, so it appears that the field equation outside always has sources. Second, there are no signals which can escape to the outside from inside the horizon, suggesting that the boundary conditions at the horizon are not capable of “monitoring” the inside of the black hole as they do for a star. As we will see momentarily, due to the large relative redshift between an asymptotic and a freely falling observer, the first difference is actually a fake, while the second is important and indeed implies the absence of hairs.

We proceed by using the “tortoise” radial coordinate r such that the (tr) part of the metric is conformally flat,

$$ds^2 = h(r)(dt^2 - dr^2) - R(r)^2 (d\theta^2 + \sin^2 \theta d\varphi^2) , \tag{2.1}$$

where

$$h(r) = 1 - \frac{R_s}{R(r)} . \tag{2.2}$$

The explicit relation between the tortoise r and the Schwarzschild R radial variables is

$$r = R + R_s \log \left(\frac{R}{R_s} - 1 \right). \quad (2.3)$$

They coincide far from the black hole at $R \rightarrow \infty$, and the tortoise coordinate $r = -\infty$ at the black hole horizon $R = R_s$. In these coordinates the scalar field satisfies the following simple wave equation,

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} + V(r) \right] (R\phi) = j, \quad (2.4)$$

with the potential given by

$$V(r) = \left(1 - \frac{R_s}{R} \right) \left(\frac{\ell^2}{R^2} + \frac{R_s}{R^3} + m_\phi^2 \right), \quad (2.5)$$

where ℓ^2 is the angular operator with eigenvalues $l(l+1)$. The form of the source j is determined by the only possible covariant coupling, $\lambda \int d\tau \phi(x_s(\tau))$, of the scalar field to the source worldline $x_s(\tau)$ parametrized by the proper time τ . As the result one obtains

$$j = \frac{\lambda}{R \sin \theta} \frac{d\tau}{dt} \delta^{(3)}(x^i - x_s^i(t)), \quad (2.6)$$

where $x^i = r, \theta, \varphi$. As the charge approaches the horizon its proper time changes more and more slowly as seen by the outside observer, and the source (2.6) extinguishes as

$$\frac{d\tau}{dt} \lesssim \sqrt{1 - \frac{R_s}{R}}.$$

We see that, just like for a star, at late times the scalar field satisfies the source-free equation outside the black hole.

The difference is that the black hole horizon, unlike the star surface, is at the infinite value $r = -\infty$ of the radial coordinate r which is a natural one for the scalar field equation (2.4). The potential $V(r)$ is shown in figure 2; it is positive everywhere outside the black hole, and vanishes near the horizon, i.e. at $r \rightarrow -\infty$. Clearly, as expected, in this situation a finite energy charge infalling into the black hole can source the scalar field outside only for a finite amount of time. Moreover, the potential (2.5) does not allow bound states with finite energy — all static solutions decaying at $r = +\infty$ diverge at the black hole horizon. Consequently, the source-free scalar field perturbations completely dissolve; they are partially absorbed by the black hole, partially radiated at infinity, and no hairs remain. Note that this conclusion is valid independently of whether the mass of the scalar field m_ϕ^2 vanishes or not.

The time scale for the loss of scalar hairs depends on details of the collapse, and in principle can be arbitrarily long as seen by the asymptotic observer. Indeed, one can take an initial scalar perturbation that follow the static solution of the scalar field equation that decays at $r = +\infty$ all the way until very large negative values of the radial coordinate r_0 (meaning very close to the black hole horizon). Such a solution will remain unperturbed in the asymptotic region on the r.h.s. of the potential barrier in figure 2 for a time of order

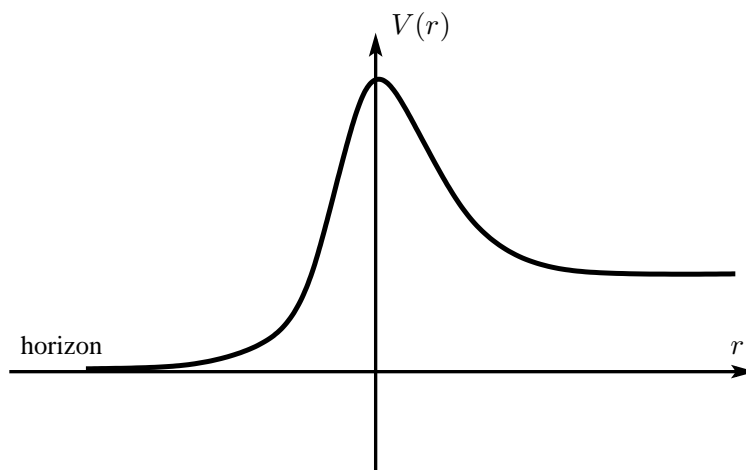


Figure 2: The potential for the scalar field perturbations in the black hole background.

$|r_0|$. Of course, this is a very fine-tuned situation; also, at fixed energy the amplitude of the scalar field goes to zero as r_0 grows. More realistically one expects that the energy of the initial scalar perturbations is concentrated not too close to the horizon, around $r \sim R_s$. Then the time scale for a decay of the scalar hairs is set by R_s .

To summarize, the loss of hairs is a two step process. First, the scalar field outside the black hole becomes source free. Second, it dissolves as a consequence of the absence of the static solutions to the perturbation equation (2.4). This is similar to how a perturbation of the sourceless free scalar field would dissolve in the infinite space.

To illustrate this picture in another example, let us consider a loss of the massive vector hairs. It is straightforward to check that for all non-spherical perturbations the situation is identical to that for the scalar field, up to extra technicalities due to more complicated tensor structure. One may expect the spherically symmetric case to be different, because at zero mass of the vector field black hole may have spherical hairs (electric and magnetic charges), so we concentrate on this case.

Two non-zero components of a spherically symmetric vector field in the black hole background (2.1) are $A_0(t, r)$ and $A_r(t, r)$. They satisfy the usual Maxwell-Proca equations

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \right) + m_A^2 g^{\nu\alpha} A_\alpha = j^\nu, \quad (2.7)$$

where the electromagnetic current j^ν has the following form for a point source,

$$j^\nu = \frac{e}{\sqrt{-g}} \delta^{(3)}(x^i - x_s^i(t)) \frac{dx^\mu}{dx^0}.$$

The only non-vanishing component of the electromagnetic strength in the spherically symmetric case is the radial electric field $E = F_{tr}$. Taking the divergence of the Proca equations (2.7) one obtains the constraint equation

$$\partial_t A_0 = \frac{1}{R^2} \partial_r (R^2 A_r). \quad (2.8)$$

Using this equation one can eliminate A_0 from the (r) component of the Proca equations and arrive at the following equation for A_r alone,

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} + V_A(r) \right] (RA_r) = e\dot{r} \left(1 - \frac{R_s}{R} \right)^{1/2} \frac{\delta^{(3)}(x^i - x_s^i(t))}{R \sin \theta}, \quad (2.9)$$

where the potential is

$$V_A = \left(1 - \frac{R_s}{R} \right) \left(\frac{2}{R^3} - \frac{6M}{R^4} + \frac{m_A^2}{R} \right).$$

Just like in the scalar case this equation becomes source-free as the charge approaches the black hole horizon. It is straightforward to check that the same is true for a mode with $A_r = 0$ and $A_0 = A_0(r)$ — the only one where the value of A_0 is not determined from the constraint equation (2.8).

So, as before, we just need to check that the massive Proca equations in the black hole background do not admit static finite energy solutions without sources. The Proca constraint (2.8) implies that $A_r \propto 1/R^2$ for a static solution, however this does not solve the wave equation (2.9) at non-zero mass. Also the (t) component of the Proca equations implies that $A_0(r)$ satisfies the time-independent Schrödinger equation with the positive potential. This completes the proof of the no-hair theorem for the massive vector field.

It is instructive to see how the massless limit is recovered. To keep this limit smooth we impose (2.8) as a gauge fixing condition. Note that this does not fix the gauge freedom completely; the residual gauge transformations are of the form $A_\mu \rightarrow A_\mu + \partial_\mu \alpha$, where α satisfies

$$\partial_t^2 \alpha - \frac{1}{R^2} \partial_r (R^2 \partial_r \alpha) = 0. \quad (2.10)$$

In the massless case the (t) component of the Proca equations reduces to the Gauss law giving the following static electric field as a solution

$$F_{\text{tr}} \propto \frac{1 - R_s/R}{R^2}.$$

The most general vector potential giving rise to this field strength and satisfying (2.8) has the form

$$A_\mu \propto \left(- \int_r^\infty dr' \frac{1 - R_s/R(r')}{R(r')^2}, 0 \right) + \partial_\mu \alpha, \quad (2.11)$$

where α solves (2.10). If $\alpha = 0$ then A_μ^2 diverges at the horizon and, consequently, in this case (2.11) does not correspond to a limit of a smooth family of solutions of the massive equations. By taking a non-trivial α one can avoid this problem and make A_μ^2 finite at the horizon, but this inevitably leads to the non-vanishing A_r component (provided one keeps A_μ^2 zero at the spatial infinity). The (r) component of the Proca equations implies then that at the non-zero photon mass there is a non-trivial time-dependence as well. This time-dependence makes a solution to dissolve on a time scale which becomes infinite as the mass is sent to zero, $m_A \rightarrow 0$ (cf. [51]).

Note that contrary to what the vector field example seems to suggest, the no-hair theorems *do not* imply that black holes are not able to support the non-trivial profile for

massive fields. For instance, a massive dilaton coupled to the photon as $\phi F_{\mu\nu}^2$ would have a non-trivial profile outside a charged black hole (similarly, a coupling of the type $\phi R_{\mu\nu\lambda\rho}^2$ would generate a scalar profile outside a neutral black hole as well). The actual content of these theorems is that the possible continuous deformations of the black hole metric in the conventional theories are related to the gauge charges, and all other details of the metric are completely determined by the latter.

To conclude, we have argued that a natural way to check the existence of hairs is to check whether the linearized field equations in the black hole background admit static finite energy solutions. We will use this procedure in the subsequent sections.

3. Higgs phases of gravity

As explained in the Introduction, we expect the presence of black hole hairs to be a generic (probably unavoidable) feature of theories with the spontaneous Lorentz violation. Analysis of the effective field theories describing gravity in the Higgs phase reveals a natural mechanism for generating the hairs: generically, the Goldstone sector of the consistent Lorentz violating models contains fields mediating instantaneous interactions. The purpose of this section is to explain the origin of these fields, and to briefly review the phenomenological constraints on these models. We also describe the exact black hole solutions relevant for the later discussion.

3.1 Setup and its basic properties

The idea behind models describing gravity in the Higgs phase is to introduce a spontaneous breaking of Lorentz invariance induced by the space-time dependent condensates of the scalar fields. In general one has four scalar fields ϕ^0, ϕ^i ($i = 1, 2, 3$) with the following vacuum expectation values (vevs) in the ground state,

$$\phi^0 = t, \quad \phi^i = x^i. \tag{3.1}$$

In order to preserve the invariance under the space-time translations one assumes that the scalar fields have purely derivative couplings. Then the action is invariant under both space-time translations and shifts of the scalar fields. The scalar vevs in equation (3.1) break both of these symmetries, however a residual symmetry remains that is the translation accompanied by the compensating shift in the fields. As a result, equations describing dynamics of perturbations around the ground state (3.1) are invariant under the space-time translations. Similarly, we assume that the action for the scalar fields has a global $O(3)$ symmetry under which ϕ^i 's transform as a vector. The ground state (3.1) is invariant under rotation of the spatial coordinates accompanied by the global rotation of ϕ^i in the opposite direction, implying rotational symmetry for perturbations. The covariant action of the theory takes the form

$$\int d^4x \sqrt{-g} (M_{\text{Pl}}^2 R[g] + \Lambda^4 F(\partial_\mu \phi^0, \partial_\mu \phi^i, \dots)) , \tag{3.2}$$

where Λ is a UV cutoff scale and dots stand for terms with larger number of derivatives acting on the scalar fields.

There is a simple physical interpretation of such systems. Namely, let us assume for a moment that the function F does not depend on the field ϕ^0 . Then the action (3.2) can be viewed as a Lagrangian description of the homogeneous and isotropic relativistic medium (fluid/solid/jelly) coupled to gravity. In this interpretation fields ϕ^i can be considered as defining a map from the physical space to the comoving fluid space. The ground state in equation (3.1) corresponds to the unperturbed fluid, while perturbations of the fields ϕ^i describe sound waves (phonons). The field ϕ^0 also fits naturally in the fluid picture. The time dependent vev of the shift-invariant scalar field indicates the presence of the charge (Bose) condensate. In other words, the action (3.2) describes dynamics of a generic relativistic *superfluid* (supersolid, superjelly) [52, 53].

When coupled to gravity, phonons and perturbations of the Bose condensate mix with the metric perturbations and give a mass to the gravitons in a way similar to the mixing of the Higgs and gauge boson perturbations in the conventional Higgs mechanism. This is true, of course, for an arbitrary fluid. However, ordinary fluids have a non-vanishing energy-momentum tensor already in the ground state. As a result, the space-time is curved and the dynamics of the metric perturbations is affected only at the length scales of the order of the curvature radius. The key property of the fluid Lagrangians that give rise to the Higgs phases of gravity is that their energy-momentum tensor vanishes (or, more generally, has the vacuum form $T_{\mu\nu} \propto g_{\mu\nu}$) in the ground state. This allows to change the dynamics of the metric perturbations directly in Minkowski (or de Sitter) space.

The requirement of the vanishing energy-momentum in the presence of the fluid and the consistency of the low-energy effective theory (in particular, absence of ghosts and rapid instabilities) are very restrictive and allow only a very limited number of possible fluid actions. The analysis of ref. [39] implies that some of the phonon modes necessarily have the degenerate dispersion relations of the form

$$\omega^2 = 0 \quad \text{or} \quad k_i^2 = 0, \tag{3.3}$$

where ω and k are frequency and spatial momentum. Modes with the $k_i^2 = 0$ dispersion relation are the instantaneous fields; their presence is crucial for the existence of the black hole hairs.

In general, the degeneracy of the dispersion relations (3.3) are broken by the higher-derivative corrections present in the effective action (3.2). Then the dispersion relations take the form

$$\omega^2 = a \frac{k^4}{\Lambda^2}, \tag{3.4}$$

$$k^2 = b \frac{\omega^4}{\Lambda^2}, \tag{3.5}$$

where a and b are dimensionless coefficients of order one. The dispersion relation (3.4) is perfectly stable. On the other hand, the dispersion relation (3.5) implies that the kinetic term of the corresponding mode is higher derivative in time. This inevitably leads to the catastrophic classical and ghost instabilities within the domain of validity of the effective theory. To exclude these instabilities one has to impose symmetries ensuring that the

modes obeying $k^2 = 0$ do not acquire time kinetic term to all orders in the derivative expansion and thus remain truly instantaneous. A minimal symmetry achieving this goal is

$$\phi^i \rightarrow \phi^i + \xi^i(\phi^0) \tag{3.6}$$

for arbitrary functions ξ^i . As a consequence of this symmetry, at the one-derivative level the function F in the action (3.2) takes the form

$$F = F(X, W^{ij}),$$

where

$$X = g^{\mu\nu} \partial_\mu \phi^0 \partial_\nu \phi^0, \tag{3.7}$$

$$W^{ij} = G^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j, \tag{3.8}$$

and the “effective metric” $G^{\mu\nu}$ is given by

$$G^{\mu\nu} = g^{\mu\nu} - \frac{\partial^\mu \phi^0 \partial^\nu \phi^0}{X} \tag{3.9}$$

In this form the origin of the instantaneous modes is very explicit; the effective metric (3.9), which determines the propagation of the ϕ^i excitations, is degenerate in the time-like direction $\partial_\mu \phi^0$. As a result, interactions mediated by the ϕ^i fields are instantaneous along the space-like surfaces of constant ϕ^0 . Excitations of ϕ^0 itself have a dispersion relation (3.4).

As shown in [54], the cosmological evolution in these models drives them into the region where an extra symmetry emerges that has the form

$$\phi^0 \rightarrow \lambda \phi^0, \quad \phi^i \rightarrow \lambda^{-\gamma} \phi^i \tag{3.10}$$

with γ being some fixed real number. One may thus impose this symmetry from the beginning. Another reason to impose this symmetry is that, as discussed later, it implies that the linearized scalar metric perturbations are described by the same equations as in GR — in particular, there is no modification to the Newton’s law. In what follows we assume that this symmetry is exact, so that at the one-derivative level the function F depends on a single combination of the scalar fields,

$$F = F(Z^{ij}), \tag{3.11}$$

where

$$Z^{ij} = X (W^{ij})^{1/\gamma}.$$

The simplest consistent Higgs phase of gravity — “ghost condensate” — corresponds to the case when the action is independent of the fields ϕ^i (loosely speaking, this corresponds to the limit $\gamma \rightarrow \infty$), so that

$$F = F(X). \tag{3.12}$$

In this case the instantaneous interactions are absent and the excitations of ϕ^0 have the dispersion relation of the form (3.4).

The symmetries (3.6), (3.10) may appear somewhat unusual. To understand better their meaning and to see how restrictive our choice of the Lorentz breaking sector (3.2) is, let us note that in the reparametrization invariant action (3.2) one can fix the “unitary” gauge in which the scalar fields are equal to their background values (3.1). In this gauge the second term in the action (3.2) takes form of the “potential” for the metric components,

$$\int d^4x \sqrt{-g} (M_{\text{Pl}}^2 R[g] + \Lambda^4 F(g_{\mu\nu}, \partial_\lambda g_{\mu\nu}, \dots)) , \quad (3.13)$$

This action explicitly breaks the diffeomorphism invariance of the theory. However, the symmetries (3.6) and (3.10) imply that in the unitary gauge the action is still invariant under the subgroup of the full diffeomorphisms generated by the time-dependent shifts of the spatial coordinates,

$$x^i \rightarrow x^i + \xi^i(t) \quad (3.14)$$

and dilatations

$$t \rightarrow \lambda t , \quad x^i \rightarrow \lambda^{-\gamma} x^i. \quad (3.15)$$

As explained in [39], an arbitrary action of the type (3.13) can be presented in the form (3.2). Consequently, our choice of the Lorentz symmetry breaking sector can be considered as a minimal one, when one does not add new degrees of freedom beyond those already present in the metric.

3.2 Phenomenology of massive gravitation

Here we review the basic phenomenological constraints on the model of ref. [40, 54].

In order for the energy-momentum tensor to have the vacuum form in the state (3.1) the function

$$f(Z) = F(Z\delta^{ij})$$

should have a minimum at $Z = 1$ (see figure 3).

The value of the cosmological constant is determined by the value $f(1)$ at the minimum. What makes the state (3.1) the true ground state (at least locally in the field space) is that the cosmological expansion drives Z to the minimum of $f(Z)$. As one approaches the attractor point $Z = 1$, the cosmological evolution is described by the conventional Friedmann equation with an extra “dark” component, parametrizing deviation from the attractor point. The equation of state of this new component is $p = w\rho$ with

$$w = -\frac{1}{3\gamma} .$$

This component has a negative pressure for $1/3 \leq \gamma \leq \infty$ and a positive pressure for $\gamma < 0$. At $0 \leq \gamma \leq 1/3$ the pressure is so negative that the null energy condition is violated. In this range of γ rapid instabilities are present and the effective field theory is not well-behaved.

The value $\gamma = 1/3$ is special. At this point the extra component behaves as the cosmological constant. What happens is that at $\gamma = 1/3$ the value of Z remains constant during the cosmological expansion, so there is no need for a function $f(Z)$ to have a

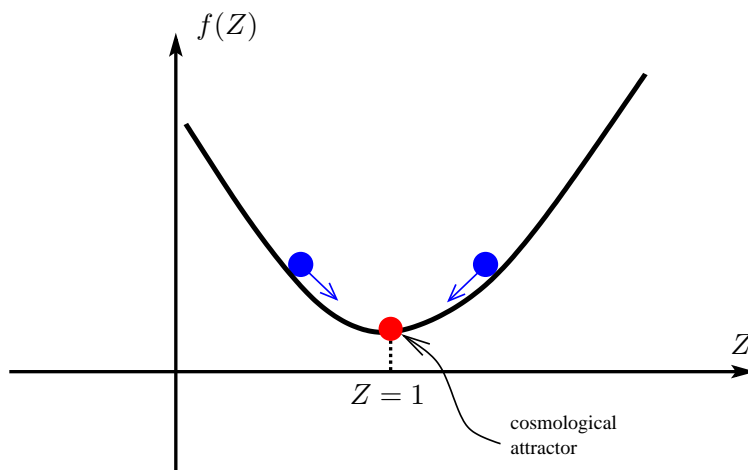


Figure 3: The shape of the kinetic function in massive gravity.

minimum. In this case the observed value of the cosmological constant is a dynamical quantity determined by the initial conditions rather than by the parameters of the action.

Let us now describe the behavior of the linearized perturbations around the Minkowski vacuum in the Higgs phase. The straightforward way to do that is to work in the unitary gauge where the scalar fields are unperturbed and a general metric perturbation takes the form

$$\begin{aligned} \delta g_{00} &= 2\varphi, \\ \delta g_{0i} &= S_i - \partial_i B, \\ \delta g_{ij} &= -h_{ij} - \partial_i F_j - \partial_j F_i + 2(\psi\delta_{ij} - \partial_i\partial_j E), \end{aligned}$$

where h_{ij} are the transverse and traceless tensor perturbations, S_i and F_i are the transverse vector perturbations, while φ , ψ , B and E are the scalar perturbations. A straightforward calculation gives that for an arbitrary energy-momentum tensor the gauge-invariant scalar potentials ψ and $\Phi \equiv \varphi + \partial_0 B - \partial_0^2 E$ are the same as in the general relativity. Similarly, for an arbitrary source the only frame-independent combination of the vector perturbations ($S_i + \partial_0 F_i$) is also the same as in the general relativity. In fact, this similarity with the general relativity persists in the case of the expanding Universe where cosmological perturbations behave in the standard way at least for some values of γ [55].

On the contrary, the transverse traceless metric perturbations h_{ij} satisfy the *massive* Klein-Gordon equation

$$(\square + m_g^2) h_{ij} = T_{ij}^{TT},$$

with the graviton mass of the form

$$m_g^2 = \alpha \frac{\Lambda^4}{M_{\text{Pl}}^2},$$

where α is a numerical coefficient of order one which is equal to a certain combination of second derivatives of the function F at $Z = 1$.

To summarize, the point important for the purposes of the current paper is that the cosmological expansion and dynamics of the linearized metric perturbations in the Higgs phases of gravity characterized by the residual symmetries (3.14), (3.15) is to a large extent the same as in the Einstein theory. This opens up a possibility for a graviton mass to be large, much larger than the present Hubble constant. The bounds on the graviton mass come from the properties of the tensor modes. The observations of the binary pulsars constrain the graviton mass to be smaller than the typical frequency of the emitted radiation,

$$m_g \lesssim 10^{-15} \text{cm}^{-1}. \quad (3.16)$$

The possibility for gravity waves to have a large mass (compared to cosmological scales) leads to a number of interesting predictions for the gravitational wave detectors. First, the graviton mass can be detected by observing a time delay between the optical and gravitational wave signal from a distant source. Second, the non-relativistic massive gravitons can be efficiently produced in the early Universe (similarly to other light scalar fields such as the axion). This would lead to a strong monochromatic line with the frequency set by the graviton mass in the stochastic gravitational wave background. This signal can be especially strong if the graviton mass is larger than $\sim 1 \text{pc}^{-1}$, so that gravitons may cluster in the Galactic halo. Interestingly, the existing limits do not exclude that all of the cold dark matter is comprised of the massive gravitons. The relevant range of frequencies is covered by the millisecond pulsar measurements and by LISA, so this possibility has good chances of being tested in the near future.

There is a more direct effect of the presence of a preferred frame (the one where the scalar vevs take the form (3.1)). Namely, the gravitational field of sources moving relative to this frame is different from that of static ones. Although the scalar modes are unaffected by the mass term, the gravitational field of a moving source has a tensor component. The non-zero mass term for the tensor modes implies that the predicted gravitational field will be different from GR. However, in practice this is an extremely small effect as the tensor component is proportional to the square of the velocity of the source relative to the preferred frame. Moreover, the effect of this component on a test mass is also proportional to the square of velocity of a test particle. At present these effects are too small to be observed.

4. Black hole backgrounds

As discussed in section 2, the most straightforward way to check whether the black hole hairs are present is to start with a known black hole solution and to check whether the spectrum of its linear perturbations contains static finite energy modes. Of course, to implement this program one has to find an explicit black hole solution first, so the main purpose of the current section is to describe the simplest black hole solutions in massive gravity. Doing this we also review some basic facts about the non-linear dynamics in these theories.

4.1 Ghost condensate

The non-linear dynamics of gravity in the Higgs phase is rather involved already in the simplest case of the ghost condensate model where the function F depends only on X . An important thing to keep in mind is that non-linear effects may not be negligible even in the regime when deviations of the metric from the Minkowski one are very small [56]. One way to understand this is to note that the field equations of the ghost condensate are equivalent to the hydrodynamical equations describing irrotational relativistic fluid with the four-velocity

$$u_\mu = \frac{\partial_\mu \phi^0}{\sqrt{X}}.$$

It is clear, therefore, that in the presence of massive objects the equations for the ghost condensate become non-linear at the time scale of order of the gravitational infall time on these objects. Naively, this makes it very hard to understand the dynamics of the model. However, the following observation leads to a radical simplification in many cases: for an arbitrary metric there exists (at least locally) a configuration of the field ϕ^0 such that

$$X = 1. \tag{4.1}$$

The field configuration that satisfies eq. (4.1) is special in many respects. First, the energy-momentum tensor of the ghost condensate ϕ vanishes at this configuration, so that there is no gravitational backreaction. Second, $X = 1$ is an attractor point of the cosmological evolution, so for many purposes $X = 1$ can be taken as a natural initial condition. Finally, if one starts with $X = 1$ this relation will continue to hold at least for some finite amount of time.

A way to understand the latter statement is to note that in the unitary gauge $\phi^0 = t$ the action of the ghost condensate model takes the form

$$M_{\text{Pl}}^2 \int d^4x \sqrt{-g} R + \Lambda^4 \int d^4x \sqrt{-g} F(g^{00}). \tag{4.2}$$

In the case of the function F having a minimum at $g^{00} = 1$ as shown in figure 3, this action can be viewed as the Einstein action with the extra term which fixes the gauge $g^{00} = 1$. Consequently, if one starts with the initial condition satisfying $X = 1$, the solution to the equations which follow from the action (4.2) will coincide with the solution to the *conventional* Einstein equations with the same initial condition, transformed into the gauge $g^{00} = 1$. This makes it clear that a large part of the non-linear dynamics of the ghost condensate is related to a question of how to transform a given solution of the Einstein equations into the gauge $g^{00} = 1$, and is irrelevant for an observer who is not directly coupled to the ghost condensate.

Apart from the situation when one initially starts with $X \neq 1$, this argument may break for the following reasons. First, the full ghost condensate action depends not only on X , but in general contains also higher-derivative scalar quantities such as $(\square \phi^0)^2$, which are suppressed by the powers of Λ . With these terms taken into account, the ghost condensate action in the unitary gauge does not have the form of the gauge-fixed Einstein action, and

the condition $X = 1$ is not preserved during the time evolution. This leads, for instance, to a slow accretion of the ghost condensate on black holes as described below. The limit on the graviton mass, eq. (3.16), implies that $\Lambda \lesssim 10$ keV. Then these effects are very slow and can be neglected on time scales of order the current age of the Universe [37, 57–59].

Second, for a given solution of the Einstein equations it is not possible in general to find a globally well-defined transformation to the gauge $g^{00} = 1$. In the fluid language this means that one expects the caustics to develop in the fluid flow where the fluid description breaks down. Accounting for these caustics leads to the space-time pictured as a patchwork of the $X = 1$ domains separated by the caustic regions where the fluid singularities are presumably resolved in the UV completed theory.

The above arguments suggest that in order to understand the dynamics of the ghost condensate (at least on reasonably short time-scales and in sufficiently small regions of space) it suffices to solve the equation $X = 1$ in a fixed Einstein geometry. For instance, a solution to this equation in the background Schwarzschild geometry takes the following form (in the Schwarzschild frame) [59]

$$\phi^0 = t + f(R), \tag{4.3}$$

where

$$f(R) = 2\sqrt{RR_s} + R_s \ln \left(\frac{\sqrt{R} - \sqrt{R_s}}{\sqrt{R} + \sqrt{R_s}} \right).$$

Since the condition $X = 1$ implies that the energy-momentum of the field configuration (4.3) vanishes, this configuration together with the Schwarzschild metric solve the equations of motion which follow from the action (4.2). In agreement with the above discussion, the scalar part (4.3) of this solution coincides with the time redefinition that transforms the Schwarzschild metric into the gauge $g^{00} = 1$.

It is straightforward to generalize this solution to the rotating case and find the Kerr black hole solution in the ghost condensate. The metric part of this solution is again the usual Kerr metric, and the ghost condensate field in the Boyer-Lindquist frame has the same general form (4.3) with a different function $f(r)$. One may be surprised that it is possible to find ϕ^0 which is independent of the angular variables in the rotating case. Note, however, that the equation $X = 1$ which one needs to solve in order to get zero energy-momentum of the ghost condensate is just the Hamilton-Jacobi equation in the Kerr background. This equation is well known, it allows for separation of variables and as a result one is able to obtain a solution in the form (4.3). The explicit form of the function $f(r)$ is somewhat more complicated in the rotating case, and we will not present it here.

4.2 Non-linearities and the simplest black holes in massive gravity

The situation is more complicated in models with gravity in the Higgs phase and massive graviton. These models possess 4 scalar fields ϕ^0 and ϕ^i which take non-zero expectation values. In order for the Schwarzschild and/or Kerr metric to be a solution of the Einstein equations these condensates have to be such that their energy-momentum tensor vanishes

in the exterior of the black hole. It is clear from the above discussion that this is possible only when the condition analogous to eq. (4.1) is satisfied, which has the form

$$Z^{ij} = \delta^{ij}. \tag{4.4}$$

As has been explained in section 3.2, there is no gravitational backreaction at this point and it is an attractor of the cosmological evolution, in full similarity with the ghost condensate model.

An important difference between eq. (4.4) and its analog in the case of the ghost condensate (4.1) is that for a generic metric eq. (4.4) cannot be solved *even locally*. Indeed, eq. (4.4) is a system of six equations which are, in general, impossible to satisfy with the four fields ϕ^0, ϕ^i .

An equivalent form of eq. (4.4) may be obtained if one goes to the unitary gauge where the action reads

$$M_{\text{Pl}}^2 \int d^4x \sqrt{-g} R + \Lambda^4 \int d^4x \sqrt{-g} F((g^{00})^\gamma g_{ij}^{-1}). \tag{4.5}$$

The second term in this action does not have a form of the gauge-fixing term. In order for the contributions of this term to the field equations to vanish the following six conditions have to be satisfied,

$$(g^{00})^\gamma g_{ij}^{-1} = \delta^{ij}. \tag{4.6}$$

These conditions are, in general, impossible to satisfy with four coordinate transformations. Note that the counting agrees with the linear analysis where only two tensor modes acquire a mass and make the extra term in the action not equivalent to the gauge fixing.

Nevertheless, it is natural to expect that at least for systems consisting of sufficiently well separated sources with small quadrupole moments (the latter requirement is not necessary if the length and time scales involved are all smaller than the inverse mass of the graviton), the qualitative picture is similar to that in the ghost condensate — one obtains a patchwork of domains where eq. (4.4) approximately holds, separated by caustic regions.

Consequently, it is reasonable to proceed similarly to the case of the ghost condensate. Namely, given a metric that solves the pure Einstein equations, one may check whether it is possible to find the scalar fields such that eq. (4.4) is satisfied, so that the metric is not modified by the backreaction of the condensates. Eq. (4.6) is an alternative form of eq. (4.4). In geometrical terms, these conditions require, in particular, that there exists a reference frame such that the metric induced on spatial slices is conformally flat. A coordinate transformation to this frame from the original one is determined by the fields ϕ^0, ϕ^i solving eq. (4.4).

A frame where the condition (4.6) holds exists for the Schwarzschild black hole and is called the Gullstrand-Painleve frame. The black hole metric in this frame is

$$ds^2 = d\tau^2 - \left(dx^i - \frac{R_s^{1/2}}{R^{3/2}} x^i d\tau \right)^2, \tag{4.7}$$

where $R = \sqrt{x_1^2 + x_2^2 + x_3^2}$. In this frame the scalar field configuration that solves eq. (4.4) is simply

$$\phi^0 = \tau, \quad \phi^i = x^i.$$

It is straightforward to check that in the Schwarzschild frame the ghost condensate part of this solution is again given by eq. (4.3), while the spatial Goldstones are $\phi^i = x^i$. Consequently, Lorentz-violating massive gravity possess a spherically symmetric black hole solution whose gravitational part is given by the Schwarzschild metric.

Let us now turn to the Kerr metric and see whether there exist solutions to eqs. (4.6) in that case. Even though the number of equations is larger than the number of unknowns, it is not obvious that all the equations are independent in this particular case. Fortunately, conformally flat spatial slicings are an important ingredient in the numerical simulations of the black hole mergers, so their existence for different solutions of the Einstein equations has been extensively studied [60, 61]. In particular, it was proven that the conformally-flat slicing of the Kerr metric is impossible due to the existence of the non-trivial invariant of the quadrupole origin [61] (loosely speaking, tensor moment)

$$\Upsilon = -112\pi J^2. \tag{4.8}$$

Note that this invariant is quadratic in the angular momentum J . Indeed, one may check by direct calculation that eqs. (4.4) for the Kerr metric can be satisfied to the linear order in J . This is a manifestation of the fact that it is not the angular momentum itself (which is the vector quantity), but rather a tensor moment that does not allow to satisfy the conditions (4.4). This is in accord with the linearized analysis of massive gravity, where only the tensor part of the metric perturbations is different from the GR case. Interestingly, the results of ref. [61] imply that not only the Kerr metric, but an arbitrary axisymmetric vacuum solution of the Einstein equations with non-zero angular momentum has a non-vanishing value of Υ and, consequently, does not allow conformally flat spatial slicings.

In view of the equivalence between eq. (4.6) and eq. (4.4), the absence of solutions to eq. (4.6) implies that there do not exist configurations of the Goldstone fields such that their energy-momentum tensor is zero in the background of the Kerr metric. Therefore, there are no solutions in massive gravity that have the Kerr metric as their metric component.

5. The toy instantaneous QED model

To understand better the meaning of the above results and to see how the instantaneous interactions that are present in massive gravity affect the no-hair theorems, it is useful to consider a simpler setup. In this section we describe a toy QED model [49, 50] which shares all the relevant features of massive gravity.

5.1 Lorentz violating electrodynamics in flat space

The flat space action for this model is

$$S = \int d^4x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 - m^2 A_i^2 \right). \tag{5.1}$$

Note that the mass term is not the standard Proca term as it only includes the spatial components of A_μ . Clearly, this mass term violates Lorentz invariance. To make the analogy with the massive gravity more explicit let us perform the scalar/vector decomposition

with respect to spatial rotations. Namely, if one writes

$$A_i = \partial_i s + a_i ,$$

with a_i being the transverse vector, $\partial_i a^i = 0$, then one obtains two decoupled sectors, the scalars (A_0, s) and the vector a_i . The scalar component of the electromagnetic field induced by an arbitrary distribution of charges is the same as in the usual electrodynamics in the gauge $s = 0$. In particular, the electrostatic limit in this model is the same as in the usual QED.

On the other hand, the vector perturbations are massive and satisfy the following equation

$$(\square + m^2) a_i = j_i^T ,$$

where j_i^T is a transverse (in the 3-dimensional sense) part of the electric current. Consequently, the electromagnetic waves acquire a mass which coexists with the long-range Coulomb potential. Note that the magnetic field is completely determined by the vector part a_i , and satisfies the usual massive Proca equation

$$(\square + m^2) B^i = \epsilon^{ijk} \partial_j j_k \tag{5.2}$$

so that no long-range magnetic field is possible. The electric field obtains contributions from both the scalar and vector sectors and satisfies the equation

$$(\square + m^2) E^i = \partial_i j_0 - \partial_0 j_i - \frac{m^2}{\partial_i^2} \partial_i j_0 . \tag{5.3}$$

An alternative way to understand some of the properties of this model is to reintroduce gauge invariance by making use of the Stückelberg trick, i.e. by replacing the vector field A_μ with the combination

$$A_\mu \rightarrow A_\mu + \partial_\mu S, \tag{5.4}$$

where S is the Stückelberg scalar field. Under a gauge transformation we now have $A_\mu \rightarrow A_\mu + \partial_\mu \chi$ and $S \rightarrow S - \chi$, so that the gauge invariance is restored.

Since the first term in the action (5.1) is gauge-invariant, the Stückelberg field enters only through the mass of the vector field. The action (5.1) does not contain a mass term for the time component A_0 , and thus the time derivative of the scalar field S does not appear in the action. Consequently, unlike in the conventional massive QED, the field S does not correspond to the new propagating degree of freedom and the action (5.1) describes only two propagating modes — the transverse components of A_i — just as in the massless case. This does not contradict to the conventional counting of degrees of freedom for Lorentz-invariant massive vector particles because the action (5.1) does not possess the symmetry allowing to go into the particle rest frame, which is necessary for the standard argument. On the other hand, there is no way to define what “transverse” means for the zero spatial momentum, so that a massive photon at rest is characterized by all three spatial components A_i in agreement with the usual counting.

The Stückelberg field enters the action only through its spatial gradients, so this field can be thought of as a kind of Lagrange multiplier. Another useful way of thinking about

this field is that it plays a role similar to the electric potential in the electrostatics, or to the gravitational potential in the Newton's theory of gravity. This suggests the existence of the instantaneous interactions in the system (5.1), and indeed the last term on the r.h.s. of eq. (5.3) gives rise to the instantaneous electric field. This does not lead to problems with causality; the existence of the preferred reference frame where the action takes the form (5.1) (in other words, the frame where the time derivatives of the Stückelberg field are absent) allows one to define unambiguous causal ordering of the events, the time ordering in the preferred frame.

5.2 Covariant action

To make the action (5.1) covariant we need to couple it to a Higgs sector that spontaneously breaks Lorentz invariance. We choose the simplest of the models of section 3.1, namely the ghost condensate model. Then the covariant form of the action (5.1) reads

$$S = \int d^4x \sqrt{-g} \left(F(X) - \frac{1}{4} F_{\mu\nu}^2 - m^2 G_\epsilon^{\mu\nu} A_\mu A_\nu \right). \quad (5.5)$$

Here X is defined in eq. (3.8) and the function F has the profile shown in figure 3. The spontaneous violation of Lorentz invariance is mediated to the vector field through the “effective metric”

$$G_\epsilon^{\mu\nu} = -g^{\mu\nu} + \epsilon \frac{\partial^\mu \phi^0 \partial^\nu \phi^0}{X} \quad (5.6)$$

where ϵ is a parameter varying between 0 and 1. When $\epsilon = 1$ this action reproduces eq. (5.1) in the Minkowski vacuum $\phi^0 = t$. This is the value we are interested in (cf. effective metric (3.9) for ϕ^i fields in massive gravity). Note that the choice $\epsilon = 1$ is protected by a residual gauge symmetry with the parameter of the gauge transformation being constant on the hypersurfaces of constant ϕ^0 ,

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(\phi^0(x)).$$

If now we introduce the Stückelberg field S according to the relation (5.4), this field will propagate in the effective metric $G^{\mu\nu}$ just as the fields ϕ^i do in massive gravity. Thus, the Lorentz-violating electrodynamics can be viewed as a theory of a single instantaneous scalar field whose shift symmetry is gauged.

5.3 Black Holes

Let us discuss what happens with the simplest black hole solutions in the model (5.5). Following the logic of section 4, let us check whether the conventional black hole solutions are preserved in the presence of the Lorentz-violating photon mass.

Clearly, zero charge black hole solutions with $A_\mu = 0$ are the same as in the pure ghost condensate model. In particular, this is the case for the neutral spherically symmetric black hole solution. As has been explained in section 4.1, the metric of this solution is the usual Schwarzschild metric, while the ghost condensate field in the Schwarzschild frame takes the form (4.3). Similarly, the neutral Kerr black hole is also a solution in the Lorentz violating QED.

Let us see now what happens with the charged spherically symmetric black holes. They have vanishing magnetic field, so it is natural to expect that it should be possible to find the usual Reissner-Nordstrom solutions in the Lorentz violating QED as well, as the electric field remains massless in this case. Indeed, as in section 4.1, we can satisfy the equation $X = 1$ for the ghost condensate with the ansatz (4.3) in the Schwarzschild-like frame where the Reissner-Nordstrom metric has form

$$ds^2 = h(R)dt^2 - h(R)^{-1}dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (5.7)$$

The vector field in this frame is equal to

$$A_\mu = \left(\frac{Q}{R}, 0, 0, 0 \right) + \partial_\mu \alpha, \quad (5.8)$$

where Q is the electric charge. Let us show that it is possible to choose the parameter of the gauge transformation α such that contributions of the photon mass term to the Maxwell, Einstein and ghost field equations vanish. The contribution to the Maxwell equations vanishes if

$$G_\epsilon^{\mu\nu} A_\nu = 0. \quad (5.9)$$

At $\epsilon = 1$ the metric $G_\epsilon^{\mu\nu}$ is positive-semidefinite with one zero eigenvector $\partial_\mu \phi^0$, so to solve (5.9) it is enough to find α such that

$$A_\mu = a \partial_\mu \phi^0 \quad (5.10)$$

for some function a . Making use of eqs. (5.10) and (4.3) one finds that the gauge parameter

$$\alpha(R) = \int dR \frac{Q}{R} f'(R)$$

does the job. It is straightforward to check that for the vector field of the form (5.10) the contributions of the mass term to the Einstein and ghost condensate equations of motion vanish as well. So, in accord with the intuitive expectation the charged spherically symmetric black hole preserves the Reissner-Nordstrom form in the Lorentz violating massive QED.

The case of a rotating charged black hole is fundamentally different. Indeed, the standard Kerr-Newman solution has non-zero electric and magnetic fields. In particular, its magnetic dipole moment is equal to

$$\mu = qJ/M, \quad (5.11)$$

where q is the electric charge. Lorentz violating massive QED does not allow long-range magnetic fields, so it does not possess rotating charged black holes with the same metric and electromagnetic field as for the Kerr-Newman black hole. A more formal way to say this is to note that for the Kerr-Newman metric both (pseudo)scalar invariants of the electromagnetic field, F^2 and $F\tilde{F}$, are non-zero. On the other hand, in order for the mass contribution to the Maxwell equations to vanish one needs the relation (5.10) to hold. It is straightforward to check that this relation implies $F\tilde{F} = 0$.

We see that the properties of the Lorentz violating QED, and in particular the fate of the conventional black hole solutions nicely match with what we found in massive gravity. A rotating black hole carries a long-range tensor component of the gravitational field (quadrupole moment), while a charged rotating black hole possesses a long-range magnetic field. As a result, these solutions are modified when the gravitational tensor mode and the magnetic field acquire a mass. Note that unlike the conventional (Lorentz-invariant) massive electrodynamics where charged black holes are absent, the black hole solutions in the Lorentz-breaking models described above should survive as they are still labeled by the conserved (due to the residual gauge symmetries) quantities, the angular momentum and charge. It is only the massive components of the corresponding solutions which get suppressed far from the black hole (similarly to the massive dilaton example mentioned in the end of section 2).

We will discuss phenomenological implications of this result in section 7. Now we are ready to address the uniqueness of these solutions and the fate of the no-hair theorems in the presence of the instantaneous interactions.

6. Instantaneous interactions and black hole hairs

It would be really surprising for no-hair theorems to hold in the presence of instantaneous interactions. Naively, one expects that in that case the black hole horizon is no longer a special place, and the information is not lost after the collapse. We will see in this section that this intuition is perfectly right. We do not consider the full theory of massive gravity to avoid unnecessary technical complications. Instead, we start by analyzing a single instantaneous scalar field in the black hole background. This example allows us to understand completely the underlying causal structure. It makes it clear that black holes indeed have an infinite amount of hairs whenever such a field is present. As a concrete example of this phenomenon in a situation close to one in massive gravity we consider neutral non-spherically symmetric black holes in the Lorentz violating electrodynamics of section 5. In particular, we demonstrate the existence of hairs which can be interpreted as the electric dipole moment of a black hole.

6.1 Instantaneous scalar field

Consider a scalar field ϕ interacting with the ghost condensate and as a result propagating in the effective metric (5.6),

$$S = \int d^4x \sqrt{-g} G_\epsilon^{\mu\nu} \partial_\mu \phi \partial_\nu \phi . \tag{6.1}$$

Eventually, we are interested in the case $\epsilon = 1$ when the field becomes instantaneous, similarly to the Goldstones ϕ^i in massive gravity. However, it is instructive to keep ϵ general for the time being. Just like for ϕ^i 's the choice $\epsilon = 1$ is protected by a symmetry $\phi \rightarrow \phi + \xi(\phi^0)$, where ξ is an arbitrary function. The field equation following from the action (6.1) is

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} G_\epsilon^{\mu\nu} \partial_\nu \phi) = j, \tag{6.2}$$

where j is an external current. For $\epsilon < 1$ the determinants of the metrics $g_{\mu\nu}$ and $G_{\epsilon\mu\nu}$ are related as

$$g = (1 - \epsilon)G_\epsilon$$

so that eq. (6.2) can be rewritten as

$$\square_G \phi = j, \tag{6.3}$$

where \square_G is the d’Alambertian defined with respect to the metric $G_\epsilon^{\mu\nu}$. In the limit $\epsilon = 1$ the metric $G_\epsilon^{\mu\nu}$ becomes degenerate and eq. (6.3) acquires a simple physical meaning. Namely, on the space-like hypersurfaces of constant ϕ^0 the operator \square_G is just a 3-dimensional Laplacian with respect to the induced metric. Consequently, at $\epsilon = 1$ eq. (6.2) is saying that on the hypersurface $\phi^0 = \text{const}$ the field ϕ coincides with the instantaneous “Newton-like” potential induced by the scalar charge distributed on this surface.

In the flat-space ghost condensate vacuum, the wave equation (6.3) describes propagation of the field with the sound velocity

$$c_\phi^2 = \frac{1}{1 - \epsilon}.$$

In the black hole background (4.7) the time redefinition $\tau \rightarrow \tau(1 - \epsilon)^{1/2}$ brings metric $G_\epsilon^{\mu\nu}$ to the same Gullstrand-Painleve form (4.7) with a different value of the Schwarzschild radius

$$\tilde{R}_s = (1 - \epsilon)R_s.$$

Intuitively, this result is very natural: the horizon area is larger for subluminal fields ($\epsilon < 0$) and smaller for superluminal ones ($\epsilon > 0$), see figure 4 (the possibility to look beyond the black hole horizons with a single derivatively coupled scalar field was discussed recently in [62]).

As $\epsilon \rightarrow 1$ the effective Schwarzschild radius \tilde{R}_s goes to zero, but also the time redefinition becomes singular. At $\epsilon = 1$ the effective metric takes form

$$G^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & -\delta^{ij} \end{pmatrix},$$

so that (6.3) takes the form of the flat-space Poisson equation on the surfaces $\tau = \text{const}$, in agreement with what we were saying before. The black hole singularity corresponds to the origin of the spatial slices. Note that the scalar field equation is not singular there.

We see that already for the superluminal field (for $0 < \epsilon < 1$) there is a sense in which black hole may have hairs. Indeed, sources inside the ordinary $c = 1$ horizon, but outside the horizon for the superluminal field (shaded region in Fig 4) are capable of supporting a static non-singular scalar profile outside the $c = 1$ horizon. Of course, these sources themselves should be superluminal in order to do this as conventional matter necessarily falls down into the black hole singularity after crossing the $c = 1$ horizon. Still this kind of hairs are not enough to resolve the thermodynamical paradoxes of refs. [45, 48] — the “*perpetua mobilia*” described there do not require any matter to be left between the two horizons.

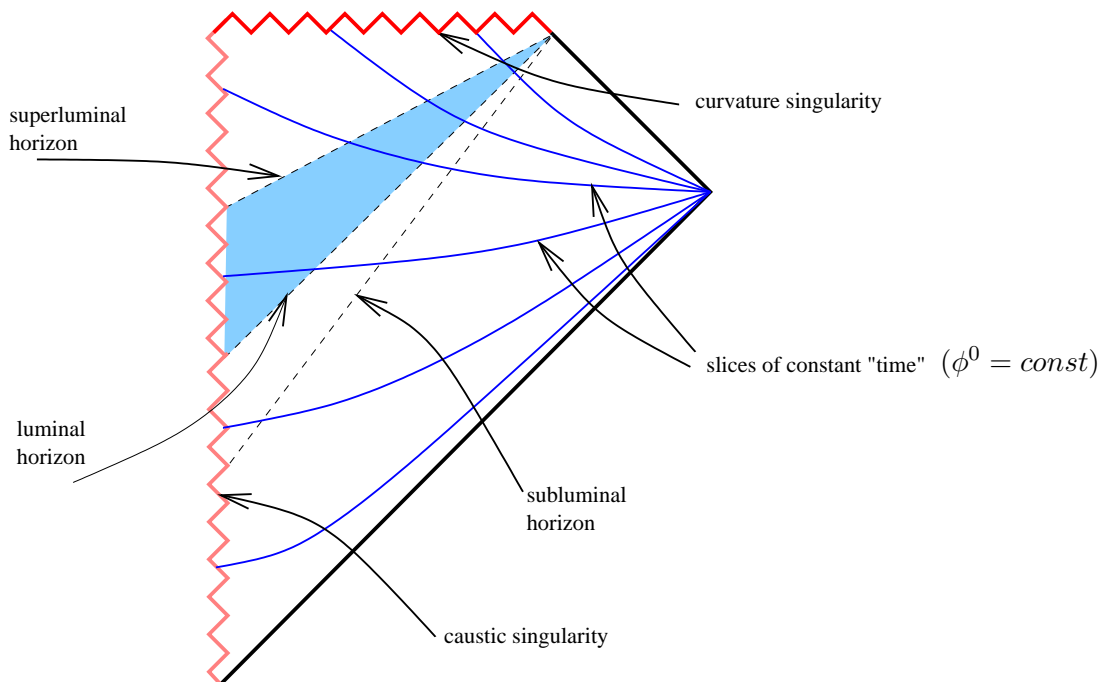


Figure 4: Penrose diagram for the black hole formed as a result of collapse in a Lorentz violating model. Blue solid lines correspond to the constant values of the ghost condensate field; instantaneous interactions make all point on these lines causally connected to each other. Note, that even before collapse there is a caustic singularity at the origin (center of the collapsing star). Dashed lines show the apparent horizons for fields with different propagation velocities.

In the instantaneous case there is no need for any sources to be present all the way until the black hole singularity to support a static scalar hair. Infinitely many kinds of hairs are possible depending on the source or, equivalently, on the boundary condition at the singularity. Of course, in general the scalar field constituting the hairs diverges as it approaches the singularity. But, unlike the conventional case where it happens at the horizon, in the case of instantaneous interactions this divergence is localized at the singularity which is present in any case, so there is no reason to require the scalar field to be regular there. Eventually, one hopes that the singularity is resolved in the UV-completed theory. This type of hairs probe the whole black hole interior and, consequently, are capable to restore the second law of thermodynamics if they “grow up” in the course of the processes described in [45, 48].

Definitely, this sensitivity of the Lorentz violating models to the boundary conditions at the black hole singularity is not an extremely appealing property, especially when compared to conventional GR where cosmic censorship cautiously prevents the asymptotic observers to face the singularity. The suggested way to eliminate the thermodynamical paradoxes may also appear brutal for a person aware of the remarkable successes of the black hole thermodynamics in GR. On the other hand, the sensitivity to the boundary conditions at the short distance singularities is a rather generic property of the non-linear solutions

in the effective theories, and GR is an exception in this respect. In addition, it is quite common in gauge theories that the same phenomenon (information recovery from the black hole in our case) appears rather differently in the Higgs phase as compared to the phase with unbroken gauge symmetry.

Finally, from the point of view of the external observer who only measures the field outside of the black hole horizon and does not directly probe the black hole interior, the Lorentz-violating black holes are not very different from usual stars. Indeed, just like in the conventional case, he sees that sources disappear as they approach the horizon and then the external field relaxes to some stationary configuration depending on the details of the collapse. Such an observer is not forced to think about singularity, just like for a star he can assume that the outside field is supported by some smooth bumpy distribution of matter inside.

At any rate, the above picture seems to be enforced in the Lorentz violating models by the very general thermodynamical arguments, and unless it is proven that these models have no chances of being UV completed, the best one can do is to study where they lead us.

6.2 Lorentz-violating QED

There is no conceptual difficulty in extending the above results to the case when the instantaneous scalar field arises as a Goldstone of the spontaneously broken gauge or reparametrization symmetry, as it happens in the Lorentz violating QED and massive gravity. Let us see how it works in practice in the technically simpler QED setup. The Lorentz violating analogue of the Proca equation takes the following form

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F^{\mu\nu}) + m_A^2 G^{\mu\nu} A_\mu = j^\nu . \tag{6.4}$$

The black hole hairs are particularly easy to identify in the limit of the large photon mass. In the leading order in the $1/m_A$ expansion the vector field should be of the form $A_\mu = a\partial_\mu\phi_0$. Then the only equation to be satisfied is a projection of the Proca equations on the $\partial_\mu\phi^0$ direction, which does not contain a mass term. As a result one obtains the following equation for the scalar function a ,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}G^{\mu\nu}\partial_\nu a) = \rho \tag{6.5}$$

where $\rho = J^\nu\partial_\nu\phi^0$. This equation is identical to the equation for the instantaneous scalar field of section 6.1 and, consequently, in the infinite mass limit Lorentz violating electrodynamics describes the instantaneous Coulomb force mediated along the space-like slices of constant ϕ^0 . In the black hole case these slices are shown in figure 4, and in the same way as for the scalar field, there is an infinite amount of the black hole hairs depending on the sources in the black hole interior (in particular, at the origin of the ϕ^0 slices — the black hole singularity).

At the finite photon mass these hairs get “dressed” by the non-vanishing Proca part of the vector field. Let us see how it happens in a simple example. Note first that the

spherically symmetric part of the vector field is entirely determined by the electric charge just like in the massless case. Indeed, the spherically symmetric field has only two non-zero components, A_0 and A_r . In the static case and in the absence of sources the r -component of the modified Proca equations (6.4) gives $G^{r\nu}A_\nu = 0$. The (tr) part of the effective metric $G^{\mu\nu}$ is degenerate, so this equation implies also $G^{t\nu}A_\nu = 0$. Hence, in the spherically symmetric case Proca equations (6.4) are equivalent to conventional massless Maxwell equations with the gauge fixing condition $A_\mu = a\partial_\mu\phi_0$. Consequently, there are no charged spherically symmetric black holes that are different from the conventional Reissner-Nordstrom ones. In other words, there are no hairs in the spherically-symmetric sector.

To provide an example of the exotic electromagnetic hairs let us consider the dipole ($l = 1$) perturbations of the vector field around a Schwarzschild black hole. In order to make this analysis parallel to the discussion of the no-hair theorems of the section 2, let us work in the tortoise coordinates. We are not changing dynamics of the sources, so that at late stages of the collapse one is again solving the empty space equations outside the black hole. Let us check that, unlike the Lorentz invariant case, there is a nonsingular static solution of the modified Proca equations (6.4) for the $l = 1, m = 0$ vector perturbations in the black hole metric (2.1). We leave the technical details for the appendix B and just outline here the main steps and results of the calculation. A static $l = 1, m = 0$ vector field configuration can be written in the form

$$A_0 = y(r)Y_2^0, \tag{6.6}$$

$$A_i = v(r)V_{2i}^0 + w(r)W_{2i}^0 + x(r)X_{2i}^0, \tag{6.7}$$

where $i = r, \theta, \phi$, while Y_l^m, V_l^m, W_l^m and X_l^m are the scalar and three vector spherical harmonics, respectively. It is immediate to check that $x(r)$ does not mix with other variables and satisfies the same equation as in the standard Proca case. Consequently, there are no black hole hairs involving non-trivial $x(r)$.

There is no need to explicitly solve the equations for the remaining functions $y(r), v(r)$ and $w(r)$ to demonstrate the existence of a regular solution; instead, one can simply count modes. Namely, the equations determining functions $y(r), v(r)$ and $w(r)$ can be rewritten in terms of a single fourth order linear equation. Therefore, an arbitrary solution is parametrized by four real parameters. One of these parameters is an overall normalization, so we are left with three. The equations are easy to solve both in the asymptotically flat ($r \rightarrow +\infty$) and in the near-horizon ($r \rightarrow -\infty$) regions. In the asymptotically flat region one finds two decreasing and two growing solutions; the requirement that the growing parts are absent leaves only one parameter. Finally, near the horizon one finds three regular and one singular solution, so one may use the remaining parameter to obtain the static solution regular at the horizon and decaying at infinity — the dipole hair.

To see how this is different from the massless case in which the hairs are absent, let us see how a similar counting works in that case. As in the Lorentz invariant case, to have a smooth massless limit we impose the analogue of the Proca constraint following from (6.4),

$$\partial_\mu (\sqrt{-g}G^{\mu\nu}A_\nu) = 0, \tag{6.8}$$

as a gauge fixing condition. Then one again finds four-parameter family of solutions, and one of the parameters is an overall normalization. Out of these solutions one is singular at the infinity, and one at the black hole horizon. So, naively one is left with two independent static solutions, the would-be hairs. However, the gauge condition (6.8) does not fix the gauge completely: there is a residual gauge freedom resulting in two pure gauge solutions among the perturbations of the type we are considering. The would-be hairs found above happen to be a gauge artifact in the massless limit. This shows that there are no physical dipole hairs of the Schwarzschild black hole in the conventional Einstein-Maxwell system.

To conclude, in this section we demonstrated the mechanism of generating the black hole hairs in theories with instantaneous interactions at the linear level in the field perturbations. In principle, one can go to higher orders of perturbation theory and calculate the backreaction of these hairs to the black hole geometry. This procedure is completely analogous to the perturbative reconstruction of, for example, the Reissner-Nordstrom metric in the limit of a small electric charge starting from the Schwarzschild metric. As in the latter case, we see no reasons for such a perturbative expansion to break down, and consequently we believe that the presence of the static hairs at the linearized level indicates the existence of the exact hairy solutions. Nevertheless, it would be of interest (both from the theoretical and phenomenological point of views) to confirm the existence of such solutions by explicit examples, either analytical or numerical.

7. Hair nurturing

So far we argued that the coexistence of the spontaneous Lorentz violation and thermodynamics strongly suggests the presence of an infinite amount of black hole hairs, and identified the source of hairs in a large class of Lorentz violating models. However, in order for these hairs to be relevant for the astrophysical observations there should be a mechanism to create them during the astrophysical collapse. For instance, the conventional family of charged black hole solutions is highly unlikely to have any observational significance as it is close to impossible to imagine how an electrically charged astrophysical black hole could have been created or could survive as such. Let us discuss the possible scenarios of how Lorentz violating hairs discussed above could have been generated. To achieve this, one has to couple an instantaneous field to conventional matter, so that it can source hairs during the collapse.

In principle, one can introduce a direct coupling of the form

$$S_{\text{direct}} = \int d^4x \sqrt{-g} G^{\mu\nu} \partial_\mu \phi J_\nu, \tag{7.1}$$

where J_ν is an arbitrary matter current. This form of the coupling preserves the symmetry $\phi \rightarrow \phi + \xi(\phi^0)$. We will not study this possibility here and just mention that the most general dimension three current one can write with the Standard model fermions is of the form

$$J^\mu = \Sigma c_i \bar{\psi} \bar{\sigma}^\mu \psi$$

and the derivative coupling of the type (7.1) to conventional Goldstone bosons gives rise to a spin dependent $1/r^3$ force. This is not the whole story as the Lorentz violating metric

$G^{\mu\nu}$ is present in the coupling (7.1); this is likely to give rise to the spin-independent, but velocity-dependent force as well (recall that there is a preferred reference frame where $\phi^0 = t$).

Given that the instantaneous fields appear as the Goldstones of the spontaneously broken space diffeomorphisms, it is natural to consider what happens in the massive gravity models of section 3.1 where the direct interactions of the instantaneous fields with matter are absent. By analogy with the case of the large photon mass considered above we expect that in the regime when the size of the black hole is large compared to the inverse graviton mass, the tensor component of the black hole metric is absent and the scalar and vector parts are completely non-universal as they are determined by the details of the collapse dynamics (or, equivalently, by the boundary conditions at the black hole singularity).

Note, however, that the binary pulsar bound on the graviton mass (3.16) implies that this regime is realized only for the black holes with mass equal (a few) $\times 10^9$ Solar masses. Such black holes are expected to exist only in the centers of the largest galaxies; a typical mass range for the galactic black holes is $10^5 - 10^7$ Solar masses. Additional problem with observing the multipole moments of these largest black holes is that it requires detecting gravity waves of low frequencies, i.e. in the range where the LISA sensitivity is worse.

So, the opposite regime when the size of the black hole is small compared to the inverse graviton mass appears to be more relevant for observations. In this case one expects hairs to be suppressed simply because the Lorentz violating massive gravity models were designed to have a smooth massless limit, and the hairs are absent at the zero graviton mass. It is instructive to see how this suppression works. Let us again consider the toy QED model and see how electromagnetic hairs emerge during the collapse of the bumpy charge distribution into the preexisting spherically symmetric neutral black hole in the limit of a small photon mass (for simplicity we assume that the collapsing distribution has zero total charge). To this end let us rearrange the modified Proca equation (6.4). Namely, let us apply the covariant derivative ∇^ρ to both sides of eq. (6.4) and antisymmetrize with respect to ρ and ν . The resulting equation can be written in the following form:

$$\mathcal{D}^2 F^{\nu\rho} + m_A^2 F^{\nu\rho} - m_A^2 \partial^{[\rho} \left(\partial^{\nu]} \phi^0 \partial^\mu \phi^0 X^{-1} A_\mu \right) = j^{[\rho\nu]} \tag{7.2}$$

where

$$j^{[\rho\nu]} = \nabla^\rho j^\nu - \nabla^\nu j^\rho .$$

In the flat space the first term in (7.2) is simply ∂_μ^2 , while in the curved background it also contains the “mass” terms proportional to the curvature. When the inverse photon mass is large compared to all other scales in the problem one can suppress the last term on the l.h.s. of eq. (7.2). Then the electromagnetic field strength is the same as in the usual Proca case (actually, in this approximation it is also the same as in the pure Maxwell theory). In particular, it vanishes outside of the black hole on the time scale of order R_s after the collapse. Consequently, at this order one can write

$$A_\mu = A_\mu^{\text{Pr}} + \partial_\mu \alpha , \tag{7.3}$$

where A_μ^{Pr} solves the conventional Proca equation (2.7) with the same charge distribution and photon mass. To find the “gauge” part α let us plug the vector field in the form (7.3)

into the constraint equation (6.8) of the Lorentz violating QED. One obtains

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}G^{\mu\nu}\partial_\nu\alpha) = -\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}G^{\mu\nu}A_\nu^{\text{Pr}}). \quad (7.4)$$

The source for α in the r.h.s. of this equation does not vanish because the constraint equation (6.8) is different from the standard Proca constraint satisfied by A^{Pr} . It does vanish outside the black hole as does A_μ^{Pr} . However, the operator acting on α in eq. (7.4) has a familiar instantaneous form, so it is enough to have a source inside the black hole to generate a non-zero α outside. This is just the same mechanism of generating hairs as discussed before. When plugged back into eq. (7.2) these α -hairs source the electromagnetic tensor outside the black hole as well. Schematically one can write

$$F^{\nu\rho} = \frac{m_A^2}{\mathcal{D}^2 + m_A^2} \partial^{[\nu}\phi^0\partial^{\rho]}(\partial^\mu\phi^0\partial_\mu\alpha). \quad (7.5)$$

Here one can estimate \mathcal{D}^2 as

$$\mathcal{D}^2 \sim \frac{1}{l^2},$$

where l is a typical length/time scale in the problem. At long scales $l \gg m_A^{-1}$ there is no suppression; this is in agreement with the flat space analysis [50] that turning on a source at a given point of space gives rise to the electric field everywhere in space, which after the time $\sim m_A^{-1}$ is suppressed only by a distance to the source (this geometric suppression is coming from solving (7.4)). In the context of the black hole hairs this implies that far away from the black hole the *relative* strength of the non-universal multipoles is unsuppressed. However, at distances l from the black hole much shorter than m_A^{-1} there is an extra suppression by a factor of $(m_A l)^2$. Note that the above argument provides a way to see the presence of the hairs which is complementary to the direct calculations of section 6.2.

The same picture is likely to hold for massive gravity as well. For small black holes we expect a suppression of the black hole hairs by the factor of $(m_A l)^2$, where l is the distance to the black hole. It is hard to reliably estimate the amplitude of the hairs within the effective field theory because it depends on the boundary conditions at the singularity. It is therefore likely that one would need the microscopic theory to calculate it quantitatively.

An estimate of the amplitude of the hairs can be provided by the following argument. Just before the collapse the metric is that of a star of the size $\sim R_s$ and of the mass $\sim M_{\text{bh}}$. The largest value of the n -th mass or angular momentum multipole of such a star is bounded by

$$M_n, J_n \lesssim M_{\text{bh}} R_s^n. \quad (7.6)$$

Assuming that after the collapse the dynamics inside the black hole does not significantly change the multipoles of the energy-momentum tensor, the above argument indicates that at large distances from the black hole $l \gg m_A^{-1}$ the part of the metric corresponding to these multipoles will remain unchanged, while at short distances $l \ll m_A^{-1}$ this part will be radiated away resulting in the suppression of the hairs by the factor $(lm_A)^2$.

This is a natural generalization of the usual bound on the angular moment of the black hole $J < M_{\text{bh}} R_s$. It follows from the intuition that the multipoles are supported

(due to the presence of the instantaneous interactions) by the conventional matter of mass M_{bh} collapsed behind the black hole horizon of size R_s . It would be interesting to check whether this bound is indeed satisfied for the gravitational collapse in massive gravity. Note, however, that higher multipoles do not correspond to any conserved charges, so in principle the dynamics inside the horizon can violate the above estimate leading to either the additional suppression of the hairs or to larger values of the multipoles. In principle, it is not clear even that the bound $J < M_{\text{bh}}R_s$ for the angular momentum needs to be satisfied in the Lorentz violating models, especially taking into account that perturbations around Lorentz violating vacuum may carry negative gravitational energy. So the verification of this bound will already be quite a non-trivial check of the GR predictions.

If the conservative bound (7.6) is correct, it makes it really challenging to observe hairs of the galactic black holes of a typical size of $10^5 - 10^7 M_\odot$ at least in the minimal massive gravity models described in section 3.1. Note, however, that the limitation comes from the bound (3.16) on the mass of the tensor gravitational waves which happens also to control the size of the hairs. One may expect that this property is not generic, and hairs can be prominent in more general models involving, for instance, extra light fields in the Lorentz-breaking sector. In principle, even in the minimal model discussed here there is a room for a larger effect. Indeed, so far we assumed that there is a single mass scale Λ in the symmetry breaking sector (see, eq. (3.2)). However natural, this assumption could easily be avoided by tuning the function F in (3.2) to give graviton a mass which is much smaller than the natural value $\sim \Lambda^2/M_{\text{Pl}}$. In any case, one potentially useful lesson from studying the minimal model is that observations of the largest black holes, and at the largest possible distance from the black hole are likely to provide the strongest constraints. This is not surprising given that it is an IR modification of gravity which gives rise to the black hole hairs.

8. Concluding remarks

To summarize, we provided a general thermodynamical argument indicating that consistent microscopic theories spontaneously breaking Lorentz invariance, if they exist, are most likely to violate the black hole no-hair theorems by allowing an *infinite* amount of black hole hairs. In particular, there is no reason in these theories to expect the higher multipole moments of the black hole metric to be universal. We identified a mechanism to generate the hairs in a broad class of theories describing gravity in the Higgs phase, which relies on the possibility to “see” inside the black hole horizon due to the presence of the instantaneous fields in the gravitational Higgs sector. In the minimal model this effect is likely to be suppressed for the typical galactic black holes due to the limit on the graviton mass which is controlled by the same parameter. This suppression may disappear for black hole masses approaching $10^9 M_\odot$. It would be interesting to understand whether this limitation can be avoided in more general models (e.g. in the bi-metric models [63, 64]).

A peculiarity of the scenario discussed here is that the black hole metric depends on the boundary condition at the singularity, and thus is in principle sensitive to quantum gravity effects. Consequently, in this class of models black holes literally provide a probe

of quantum gravity. This is not so unusual for non-linear solutions in the effective theories, a notable exception being the conventional GR which provides a mechanism (“cosmic censorship”) to prevent large class of observers from probing the physics at singularities.

As an open question we mention that there is a subtlety in defining the multipole moments for black holes discussed here. Indeed, the conventional multipole moments are defined by assuming that the metric satisfies the vacuum Einstein equations, which is not the case for rotating black holes in massive gravity. Nevertheless, it is very likely that the tests of universality of the black hole moments assuming that the metric satisfies the vacuum Einstein equations (the conventional procedure) will be sensitive to the non-universality of the type we discuss as well. Still, it would be useful to develop a model-independent approach that would not make such an assumption.

Finally, note that the instantaneous effects used here to look behind the black hole horizons are likely to allow to look behind the cosmological horizon as well. It may be interesting to study the possible consequences of this effect in more detail.

Acknowledgments

We thank Nima Arkani-Hamed, Gia Dvali, Markus Luty, Alberto Nicolis, Raman Sundrum and Matt Schwartz for interesting and stimulating discussions. The work of P.T. is supported in part by IISN, Belgian Science Policy (under contract IAP V/27).

A. UV modifications of gravity and black holes

Let us assume that the cutoff scale in the gravitational sector is Λ . Then, in a “model independent” way one can write the covariant effective gravitational action at scales below Λ as

$$S_{\text{gr}} = \int d^4x \sqrt{-g} \left(M_{\text{Pl}}^2 R + a_1 \frac{M_{\text{Pl}}^2}{\Lambda^2} R^2 + \dots + a_{n-1} \frac{R^n}{\Lambda^{(2n-4)}} \left(\frac{M_{\text{Pl}}}{\Lambda} \right)^n + \dots \right) \quad (\text{A.1})$$

where, in general, a_n are of order one. Of course, this action contains also terms with more covariant derivatives (each covariant derivative brings an extra factor of Λ^{-1}), such as $M_{\text{Pl}}^2/\Lambda^4 \nabla_\mu R \nabla^\mu R$. The leading vertex from the n -th term in eq. (A.1) is $\sim (\partial^2 h)^n$, so the choice of $(M_{\text{Pl}}/\Lambda)^n$ in eq. (A.1) ensures that the corresponding amplitude becomes of order one at energies of order Λ . This is the largest coefficient one can put in front of R^n terms while keeping the effective theory valid up to the scale Λ .

In this model-independent scenario, there is a limit on Λ coming from the short-distance tests of the Newton’s law (for instance, from the modifications of the coupling to matter via generation of operators like $\Lambda^{-2} \int d^4x \sqrt{-g} R_{\mu\nu} T^{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor), so that Λ , in principle, can be as low as a fraction of mm^{-1} . Note, however, that these are probes of gravity in the linear regime, so there is a room for significant classical non-linear effects due to the high order terms in (A.1) at the much longer distance scales, provided the curvature is large enough.

Namely, the contribution of the n -th term in the action (A.1) to the classical equations equals to that of the first for curvatures

$$R \geq \frac{\Lambda^3}{M_{\text{Pl}}} \left(\frac{M_{\text{Pl}}}{\Lambda} \right)^{1/(n-1)}.$$

Hence the classical contribution grows with n and asymptotically the critical curvature is

$$R_{\text{crit}} \simeq \Lambda^3/M_{\text{Pl}} \simeq (10^{15} \text{ cm})^{-2} (\Lambda \cdot \text{mm})^3$$

Requiring that these non-linear effects are small at the BBN epoch would impose slightly stronger constraint on Λ than coming from short distance tests, namely one needs $R_{\text{crit}} \gtrsim (10^{11} \text{ cm})^{-2}$. In principle, this still leaves a room for new non-linear effects at the scales of the galactic black holes. Note, however, that it is just non-linearities themselves, rather than the presence of horizon, that are needed for these effects to show up. Also, these effects are much more likely to show up for smaller scale non-linear systems such as binary neutron stars and stellar mass black holes.

On the theoretical side it is extremely hard to imagine a viable model achieving such a low cutoff scale in gravity. Note that unlike massive gravity where the low cutoff is present in a separate gravitationally coupled Higgs sector, here this is gravity itself which is strongly coupled at the scale Λ . For instance, in string theory the cutoff of gravity is at the string scale $\Lambda \sim \alpha'^{-1/2}$, and above this scale an infinite tower of new gravitationally interacting particles arises. Typically this results in too strong constraints on the cutoff scale to allow non-linear effects at the scale of the galactic black holes (see ref. [14] for a discussion of the constraints on the gravitational cutoff in string-inspired scenarios).

Finally, note that the coefficients a_i in the effective action (A.1) can be parametrically smaller in a particular model. For instance, the expansion of the gravitational action following from the string theory gives

$$M_{\text{Pl}}^2 (R + \alpha' R^2 + \alpha'^2 R^3 + \dots).$$

In this case one need curvatures of order the cutoff scale in order for non-linear effect to be large.

To summarize, it appears highly non-plausible that UV effects in the gravitational sector can be important at the scales of the galactic black holes. Even if such effects were present, they would have nothing to do with the presence of the horizon. In particular, there are no reasons to expect that they would lead to the non-universality of the black hole multipoles.

B. Electric dipole hair

Here we provide some details of the calculation that shows the presence of a static deformation (hair) of the Schwarzschild black hole with $l = 1$, $m = 0$ vector field in the Lorentz violating QED. At general values of l and m the spherical vector harmonics used in eq. (6.6)

have the following form,

$$V_l^m = \left(0, -\sqrt{\frac{l+1}{2l+1}} Y_l^m, \frac{1}{\sqrt{(l+1)(2l+1)}} r \partial_\theta Y_l^m, \frac{im}{\sqrt{(l+1)(2l+1)}} r Y_l^m \right), \quad (\text{B.1})$$

$$W_l^m = \left(0, \sqrt{\frac{1}{2l+1}} Y_l^m, \frac{1}{\sqrt{l(2l+1)}} r \partial_\theta Y_l^m, \frac{im}{\sqrt{l(2l+1)}} r Y_l^m \right), \quad (\text{B.2})$$

$$X_l^m = \left(0, 0, -\frac{m}{\sqrt{l(l+1)} \sin \theta} r Y_l^m, -\frac{i}{\sqrt{l(l+1)}} r \sin \theta \partial_\theta Y_l^m \right). \quad (\text{B.3})$$

Here Y_l^m are the usual scalar spherical harmonics. For the problem at hand it is convenient to redefine the coefficient functions y, v, w in the following way,

$$\begin{aligned} y(r) &= 2\sqrt{\frac{\pi}{3}} \left(\alpha(r) - \sqrt{1-h(r)} \beta(r) \right), \\ v(r) &= \sqrt{2} \left(-2\sqrt{\pi} \left(\sqrt{1-h(r)} \alpha(r) + \beta(r) \right) + \delta(r) \right), \\ w(r) &= \frac{1}{3} \left(2\sqrt{\pi} \left(\sqrt{1-h(r)} \alpha(r) + \beta(r) \right) + 2\delta(r) \right). \end{aligned}$$

Then at $l = 1, m = 0$ the explicit form of the vector field components in terms of the functions α, β, δ and x is

$$A_\theta = \cos \theta \left(\alpha(r) - \sqrt{1-h(r)} \beta(r) \right), \quad (\text{B.4})$$

$$A_r = \cos \theta \left(\sqrt{1-h(r)} \alpha(r) + \beta(r) \right), \quad (\text{B.5})$$

$$A_\theta = -\frac{r \sin \theta \delta(r)}{2\sqrt{\pi}} \quad (\text{B.6})$$

$$A_\phi = -\sqrt{\frac{3}{8\pi}} r \sin^2 \theta x(r) \quad (\text{B.7})$$

By plugging these expressions into the modified Proca equations (2.9) one gets a set of the ordinary differential equations for the radial functions α, β, δ and x . There is no point in writing down these equations explicitly. Note first that the equation for the function x decouples from the rest and is the same as in the conventional Proca theory. So there is no hairs associated with this function and we set $x = 0$ in what follows. Using the remaining equations one can solve for α and δ in terms of β and obtain the 4th order differential equation for β alone. This equation is rather involved, but as explained in the main text, all that we are doing is the mode counting, so we need an explicit form of this equation only in the limits $r \rightarrow \pm\infty$. In the leading order in the near-horizon limit $r \rightarrow -\infty$ this equation takes form

$$R_s^4 \beta^{(4)} - 8R_s^3 \beta^{(3)} + 23R_s^2 \beta'' - 28R_s \beta' + 12\beta = 0 \quad (\text{B.8})$$

A general solution for β in the near horizon limit is

$$\beta = C_1 e^{r/R_s} + C_2 e^{3r/R_s} + C_3 e^{2r/R_s} + C_4 \left(e^{2r/R_s} (2R + R_s) + \mathcal{O}(e^{3r/R_s}) \right) + \mathcal{O}(e^{4r/R_s})$$

It is important to keep track of the order of magnitude of the subleading terms as α and δ are enhanced relative to β by a factor of e^{2R/R_s} . As a result, eq. (B.8) allows to find A_μ and $F_{\mu\nu}$ near the horizon including terms of order $\mathcal{O}(1)$ or even $\mathcal{O}(e^{r/R_s})$ if $C_4 = 0$. For instance, at the order $\mathcal{O}(1)$ the vector field at the horizon is

$$A_\mu dx^\mu = R_s e \cos \theta (eC_4 - m_A^2 R_s C_1) (dt + dr) - \frac{R_s}{2} e \sin \theta (7C_1 + 2eC_3 + 2e(r + R_s)C_4) d\theta. \quad (\text{B.9})$$

The most reliable way to check whether this field is regular at the horizon is to transform it to the Kruskal coordinates (or any other coordinate system regular at the horizon). In this way one finds that the vector field (B.9) is always regular at the horizon, while the corresponding field strength is also regular iff $C_4 = 0$. Therefore the Lorentz violating Proca equation has three linearly independent solutions that are regular at the horizon. A nice consistency check of the calculation is that in the massless limit $m_A^2 = 0$ in the order $\mathcal{O}(e^{r/R_s})$ the field strength depends only on the two independent combinations of the coefficients C_i , namely on C_4 and $(8C_1 + 2e^2C_2 + 9eC_3)$. This is in agreement with the existence of the two pure-gauge solutions $A_\mu = \partial_\mu(a(r)Y_2^0)$ satisfying the gauge condition (6.8). Note that this mode counting is different from the usual Proca case where the same procedure gives only two (instead of three) solutions regular at the horizon.

A similar analysis reveals two finite-energy solutions at the spatial infinity, one of them is the electric dipole with $E \sim 1/r^3$ and another the exponentially damped magnetic dipole with $B \sim e^{-m_A r}$. Another pair of solutions has an infinite energy — for one of them the amplitude of the electric field approaches a constant, and for the other the magnetic field grows exponentially. Consequently, the mode counting argument implies that there is only one mode which is regular both at the horizon and at infinity. This mode becomes pure gauge in the massless limit.

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